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BALANCED FACTOR CONGRUENCES ON PRE A*- ALGEBRAS

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ABSTRACT

In this paper we prove for any element a of a Pre A*-algebra A, the set A_a is a Pre A*-algebra under the operations $\wedge$ and $\vee$ and the complementation ^{a}. We also prove that if θ is a factor congruence on A, then $\theta \cap (A_a \times A_a)$ is a balanced factor congruence of A_a for each a$\in$A and hence there exists unique $s_a \in$ B(A_a) such that $\theta \cap (A_a \times A_a) = \theta_{s_a}$ = {(x, y) $\in A_a \times A_a$ / $s_a \wedge$ x = $s_a \wedge$ y}.

Keywords: Boolean algebra, Pre A*-algebra, congruence, centre of Pre A*-algebra, factor congruence, balanced factor congruence.

AMS subject classification (2000): 06E05, 06E25 ,06E99,06B10

INTRODUCTION

In 1948 the study of lattice theory had been made by Birkhoff [7]. In a draft paper [3], The Equational Theory of Disjoint Alternatives around 1989, E.G.Maines introduced the concept of Ada(Algebra of disjoint alternatives) $(A,\wedge,\vee,(-)',(-)_\pi,0,1,2)$ which is however differ from the definition of the Ada of his later paper [4] Adas and the equational theory of if-then-else in 1993. While the Ada of the earlier draft seems to be based on extending the If-Then –Else concept more on the basis of Boolean algebra and the later concept is based on C-algebra $(A,\wedge,\vee,')$ introduced by Fernando Guzman and Craig.C. Squir[1].

In 1994, P. Koteswara Rao[2] first introduced the concept A*-Algebra $(A,\wedge,\vee,*,(\tilde{-}),(-)_\pi,0,1,2)$ not only studied the equivalence with Ada, C-algebra, Ada's connection with **3-** Ring, the If-Then-Else structure over A*-algebra and Ideal of A*-algebra. . In 2000, J. Venkateswara Rao [5] introduced the concept of Pre A*-algebra $(A,\wedge,\vee,(\tilde{-}))$ as the variety generated by the 3-element algebra A = {0,1,2} which is an algebraic form of three valued conditional logic.In [8] A.Satyanarayana et al. generated Semilattice structure on Pre A*-Algebras .In [10], A.Satyanarayana defined a partial ordering on a Pre A*-algebra A and the properties of A as a poset are studied. In [9] A.Satyanarayana. et.all derive necessary and sufficient conditions for pre A*-algebra A to become a Boolean algebra in terms of the partial ordering**.**

In this paper we prove for any element a of a Pre A*-algebra A, the set A_a is a Pre A*-algebra under the operations $\wedge$ and $\vee$ induced by those of A and the complementation ^{a}. We also prove that if θ is a factor congruence on A ,then $\theta \cap (A_a \times A_a)$ is a balanced factor congruence of A_a for each a$\in$A and hence there exists unique $s_a \in$ B(A_a) such that $\theta \cap (A_a \times A_a) = \theta_{s_a}$.

1. PRELIMINARIES:

1.1. Definition: Boolean algebra is an algebra $(B, \vee, \wedge, (-)', 0, 1)$ with two binary operations, one unary operation (called complementation), and two nullary operations which satisfies:

(i) $(B, \vee, \wedge)$ is a distributive lattice

(ii) $x \wedge 0 = 0,\ x \vee 1 = 1$

(iii) $x \wedge x' = 0,\ x \vee x' = 1$

We can prove that $x'' = x,\ (x \vee y)' = x' \wedge y'$, $(x \wedge y)' = x' \vee y'$ for all $x, y \in B$.

In this section we concentrate on the algebraic structure of Pre A*-algebra and state some results which will be used in the later text.

1.2. Definition: An algebra $(A, \wedge, \vee, (-)^{\sim})$ where A is non-empty set with 1, $\wedge$, $\vee$ are binary operations and $(-)^{\sim}$ is a unary operation satisfying

(a) $x^{\sim\sim} = x \quad \forall x \in A$

(b) $x \wedge x = x, \quad \forall x \in A$

(c) $x \wedge y = y \wedge x, \quad \forall x, y \in A$

(d) $(x \wedge y)^{\sim} = x^{\sim} \vee y^{\sim} \quad \forall x, y \in A$

(e) $x \wedge (y \wedge z) = (x \wedge y) \wedge z, \quad \forall x, y, z \in A$

(f) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z), \quad \forall x, y, z \in A$

(g) $x \wedge y = x \wedge (x^{\sim} \vee y), \quad \forall x, y \in A$ is called a Pre A*-algebra

1.3. Example [6]: 3 = {0, 1, 2} with operations $\wedge, \vee, (-)^{\sim}$ defined below is a Pre A*-algebra.

$\wedge$	0	1	2
0	0	0	2
1	0	1	2
2	2	2	2

$\vee$	0	1	2
0	0	1	2
1	1	1	2
2	2	2	2

x	$x^{\sim}$
0	1
1	0
2	2

1.4. Note: The elements 0, 1, 2 in the above example satisfy the following laws:

(a) $2^{\sim} = 2$ (b) $1 \wedge x = x$ for all $x \in$ **3**

(c) $0 \vee x = x$ for all $x \in$ **3** (d) $2 \wedge x = 2 \vee x = 2$ for all $x \in$ **3**.

1.5. Example: 2 = {0, 1} with operations $\wedge, \vee, (-)^{\sim}$ defined below is a Pre A*-algebra.

$\wedge$	0	1
0	0	0
1	0	1

$\vee$	0	1
0	0	1
1	1	1

x	$x^{\sim}$
0	1
1	0

1.6. Note:

(i) $\left(2, \vee, \wedge, (-)^{\tilde{}}\right)$ is a Boolean algebra. So every Boolean algebra is a Pre A* algebra.

(ii) The Boolean algebra **2** = {0, 1} is an underlying set in the Pre A*-algebra **3 =** {0, 1, 2}. Further the readers can note that the Pre A*-algebra **3 =** {0, 1, 2} is not a Boolean algebra as there is a critical element 2 which is not in the Boolean algebra **2** = {0, 1}.

(iii) The identities 1.2(a) and 1.2(d) imply that the varieties of Pre A*-algebras satisfies all the dual statements of 1.2(b) to 1.2(g).

1.7. Definition: Let A be a Pre A*-algebra. An element x $\in$A is called central element of A if $x \vee x^{\tilde{}} = 1$ and the set {x $\in$ A/ $x \vee x^{\tilde{}} = 1$} of all central elements of A is called the centre of A and it is denoted by B (A).

1.8. Theorem:[6] Let A be a Pre A*-algebra with 1, then B (A) is a Boolean algebra with the induced operations $\wedge, \vee, (-)^{\sim}$

1.9. Lemma: [10] Let $(A, \wedge, \vee, (-)^{\tilde{}})$ be a Pre A*-algebra and let $a \in A$.Then the relation $\theta_a = \{(x, y) \in A \times A / \ a \wedge x = a \wedge y\}$ is

(a) a congruence relation as in Boolean algebra.

(b) $\theta_a \cap \theta_{a^{\tilde{}}} = \theta_{a \vee a^{\tilde{}}}$

(c) $\theta_a \cap \theta_b \subseteq \theta_{a \vee b}$

(d) $\theta_a \cap \theta_{a^{\tilde{}}} \subseteq \theta_{a \wedge a^{\tilde{}}}$

we will write $x \ \theta_a y$ to indicate $(x, y) \in \theta_a$.

1.10. Lemma: [10] Let A be a Pre A*-algebra and a, b $\in$ $B(A)$ (Boolean algebra with the induced operations $\wedge, \vee, (-)^{\sim}$), then $\theta_a \cap \theta_b = \theta_{a \vee b}$

1.11. Definition: Let A be a Pre A*-algebra and α $\in$Con(A).Then α is called factor congruence if there exist β $\in$Con(A) such that $\alpha \cap \beta = \Delta_A$ and $\alpha \ o \ \beta$ = A$\times$A. In this case β is called direct complement of α.

1.12. Definition: A congruence β on Pre A*-algebra A is called balanced if $(\beta \vee \theta) \cap (\beta \vee \theta^{\tilde{}}) = \beta$ for any direct factor congrucnces θ and any of its direct comoplement $\theta^{\tilde{}}$ on A.

2.Balanced Factor Congruences on Pre A*- algebras:

2.1. Theorem: Let A be a Pre A*-algebra and a $\in$A. Let A_a = {x$\in$A / a$\wedge$x = x } then A_a is closed under the operations $\wedge$ and $\vee$. Also for any x$\in$ A_a define, x^a =a$\wedge$ x$^{\tilde{}}$ then (A_a, $\wedge$,$\vee$,^{a}) is a Pre A*-algebra with 1(here a is itself is the identity for $\wedge$in A_a;that is 1 in A_a).

Proof: Let x,y $\in$ A_a.Then a$\wedge$x = x and a$\wedge$y = y.

Now a$\wedge$(x$\wedge$ y) = (a$\wedge$x)$\wedge$ y = x$\wedge$ y $\Rightarrow$ x$\wedge$ y$\in$ A_a

Also a$\wedge$(x$\vee$ y) = (a$\wedge$x) $\vee$(a$\wedge$y) = x$\vee$ y$\Rightarrow$ x$\vee$y$\in$ A_a

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Therefore A_a is closed under the operation $\wedge$ and $\vee$.

a $\wedge$ x^a = a $\wedge$ (a $\wedge$ $x^{\sim}$) = a $\wedge$ $x^{\sim}$ = $x^a \Rightarrow x^a \in A_a$

Thus A_a is closed under ^{a}.

Now for any x, y, z $\in A_a$

(1) $x^{aa} = (a \wedge x^{\sim})^a = a \wedge (a \wedge x^{\sim})^{\sim} = a \wedge (a^{\sim} \vee x) = a \wedge x = x$

(2) $x \wedge x = (a \wedge x) \wedge (a \wedge x) = a \wedge x = x$

(3) $x \wedge y = (a \wedge x) \wedge (a \wedge y) = (a \wedge y) \wedge (a \wedge x) = y \wedge x$

(4) $(x \wedge y)^a = a \wedge (x \wedge y)^{\sim} = a \wedge (x^{\sim} \vee y^{\sim})$

$= (a \wedge x^{\sim}) \vee (a \wedge y^{\sim})$

$= x^a \vee y^b$

(5) $x \wedge (y \wedge z) = (a \wedge x) \wedge \{(a \wedge y) \wedge (a \wedge z)\}$

$= a \wedge \{x \wedge (y \wedge z)\}$

$= a \wedge \{(x \wedge y) \wedge z\}$ (since x, y, z $\in$ A)

$= (x \wedge y) \wedge z$

(6) $x \wedge (y \vee z) = (a \wedge x) \wedge \{(a \wedge y) \vee (a \wedge z)\}$

$= \{(a \wedge x) \wedge (a \wedge y)\} \vee \{(a \wedge x) \wedge (a \wedge z)\}$

$= \{a \wedge (x \wedge y)\} \vee \{(a \wedge (x \wedge z)\}$

$= (x \wedge y) \vee (x \wedge z)$

(7) $x \wedge (x^a \vee y) = x \wedge \{ (a \wedge x^{\sim}) \vee y\}$

$= \{x \wedge (a \wedge x^{\sim})\} \vee (x \wedge y)$

$= (x \wedge x^{\sim}) \vee (x \wedge y)$ (since $a \wedge x = x$)

$= x \wedge (x^{\sim} \vee y)$

$= x \wedge y$

Finally x $\in A_a$ implies that $a \wedge x = x = x \wedge a$. Thus (A_a, $\wedge$, $\vee$, ^{a}) is a Pre A*-algebra with a as identity for $\wedge$.

2.2. Theorem: Let θ be a congruence on A. Then $\theta \cap (A_a \times A_a)$ is a congruence on A_a, for each a $\in$ A.

Proof: Suppose θ be a congruence on A, and a $\in$ A.

Let x $\in A_a$ we have that (x, x) $\in A_a \times A_a$

Since θ be congruence we have (x, x) $\in \theta \cap (A_a \times A_a)$

Therefore the result is reflexive.

Let (x, y) $\in \theta \cap (A_a \times A_a)$

Then (x, y) $\in \theta$ and (x, y) $\in A_a \times A_a$

$\Rightarrow$ (y, x) $\in \theta$ and (y, x) $\in A_a \times A_a$

$\Rightarrow$ (y, x) $\in \theta \cap (A_a \times A_a)$

The result is Symmetric.

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Let (x, y) , (y, z) $\in \theta \cap (A_a \times A_a)$

$\Rightarrow$ (x, y) , (y, z) $\in \theta$ and (x, y) , (y, z) $\in A_a \times A_a$

$\Rightarrow$ (x, z) $\in \theta$ and x, y, z $\in A_a$ hence (x, z) $\in A_a \times A_a$

$\Rightarrow$ (x, z) $\in \theta \cap (A_a \times A_a)$

The result is Transitive.

Hence the relation is equivalence relation.

Let (x, y), (z, t) $\in \theta \cap (A_a \times A_a)$

Since x, y, z, t $\in A_a$, we have

$a \wedge x \wedge z = x \wedge z \Rightarrow x \wedge z \in A_a$

$a \wedge y \wedge t = y \wedge t \Rightarrow y \wedge t \in A_a$

$\Rightarrow (x \wedge z, y \wedge t) \in A_a \times A_a$

Since θ be congruence we have $(x \wedge z, y \wedge t) \in \theta \cap (A_a \times A_a)$

Now (x, y) $\in \theta \Rightarrow (x^{\sim}, y^{\sim}) \in \theta$

$\Rightarrow (a \wedge x^{\sim}, a \wedge y^{\sim}) \in \theta$ and $(a \wedge x^{\sim}, a \wedge y^{\sim}) \in A_a \times A_a$

$\Rightarrow (a \wedge x^{\sim}, a \wedge y^{\sim}) \in \theta \cap (A_a \times A_a)$

$\Rightarrow (x^{a}, y^{a}) \in \theta \cap (A_a \times A_a)$

Therefore $\theta \cap (A_a \times A_a)$ is compatible (closed) with the binary operation $\wedge$ and unary operation ^{a} on A_a.

Let (x, y), (z, t) $\in \theta \cap (A_a \times A_a)$

Since x, y, z, t $\in A_a$, we have

$a \wedge (x \vee z) = (a \wedge x) \vee (a \wedge y) = x \vee z \Rightarrow x \vee z \in A_a$

$a \wedge (y \vee t) = (a \wedge y) \vee (a \wedge t) = y \vee t \Rightarrow y \vee t \in A_a$

$\Rightarrow (x \vee z, y \vee t) \in A_a \times A_a$

Therefore $\theta \cap (A_a \times A_a)$ is compatible with $\vee$ also

Thus $\theta \cap (A_a \times A_a)$ is a congruence relation on A_a.

2.3. Theorem: Let θ be a factor congruence on a Pre A*-algebra A. Then $\theta \cap (A_a \times A_a)$ is a factor congruence on A_a.

Proof: Since θ be a factor congruence on A there is a congruence $\theta^{\sim}$ on A such that $\theta \cap \theta^{\sim} = \Delta_A$ and $\theta \circ \theta^{\sim} = A \times A$.

Consider $[\theta \cap (A_a \times A_a)] \cap [\theta^{\sim} \cap (A_a \times A_a)] = (\theta \cap \theta^{\sim}) \cap (A_a \times A_a)$

$= \Delta_A \cap (A_a \times A_a)$

$= \Delta_{A_a}$, the diagonal on A_a

Observe that every element in A_a is the form a $\wedge$ x for some x $\in$ A.

Now, let (a $\wedge$ x ,a $\wedge$ y) $\in A_a \times A_a$ .Then (a $\wedge$ x, a $\wedge$ y) $\in$ A $\times$ A = $\theta \ o\ \theta^{\sim}$ which implies that there exist z $\in$ A such that (a $\wedge$ x ,z) $\in \theta$ and (z, a $\wedge$ y) $\in$ $\theta^{\sim}$.

Now (a $\wedge$ x, a $\wedge$ z) $\in \theta$ and (a $\wedge$ z, a $\wedge$ y) $\in$ $\theta^{\sim}$ and a $\wedge$ z $\in A_a$

Therefore (a $\wedge$ x, a $\wedge$ z) $\in \theta \cap (A_a \times A_a)$ and (a $\wedge$ z, a $\wedge$ y) $\in \theta^{\sim} \cap (A_a \times A_a)$ and hence (a $\wedge$ x ,a $\wedge$ y) $\in$ $[\theta^{\sim} \cap (A_a \times A_a)]\ o\ [\theta \cap (A_a \times A_a)]$

Therefore $[\theta \cap (A_a \times A_a)]\ o\ [\theta^{\sim} \cap (A_a \times A_a)] = A_a \times A_a$

Therefore $\theta \cap (A_a \times A_a)$ is a factor congruence on A_a and $\theta^{\sim} \cap (A_a \times A_a)$ is a direct complement of $\theta \cap (A_a \times A_a)$.

2.4. Theorem: Let A be a Pre A*-algebra with 1 induced by Boolean algebra , θ is a factor congruence on A and β a direct complement of θ.Then there exist unique a $\in$ A such that $\theta = \theta_a$ = $\{(x,y) \in A \times A /\ a \wedge x = a \wedge y\}$ and β = $\theta_{a^{\sim}}$ (= β_a).

Proof: Let $1^{\sim}$ = 0. Then 1 and 0 are identities for operators $\wedge$ and $\vee$ respectively in A.

We have $\theta \cap \beta = \Delta_A$ and $\theta\ o\ \beta$ = A $\times$ A.

Then clearly $\theta\ o\ \beta = \beta\ o\ \theta$ = A $\times$ A.

Since (0, 1) $\in$ A $\times$ A = $\theta\ o\ \beta$, there exist a $\in$ A such that (0, a) $\in \beta$ and (a, 1) $\in \theta$.

First we observe that a is a unique element with the above property. If b $\in$ A also is such that (0, b) $\in \beta$ and (b, 1) $\in \theta$ then by the transitive and symmetry of β and θ we get (a, b) $\in \theta \cap \beta = \Delta_A$, the diagonal of A, and hence a = b

Thus a is unique such that (0, a) $\in \beta$ and (a, 1) $\in \theta$

Now we prove that $\theta = \theta_a$ and $\beta = \theta_{a^{\sim}}$

For any x, y $\in$ A we have

(0, a $\wedge$ x) = (0 $\wedge$ x, a $\wedge$ x) $\in \beta$ (since (0, a) $\in \beta$) and hence (a $\wedge$ x, a $\wedge$ y) $\in \beta$

Now (x, y) $\in \theta \Rightarrow$ (a $\wedge$ x, a $\wedge$ y) $\in \theta \cap \beta = \Delta_A$

$\Rightarrow$ a $\wedge$ x = a $\wedge$ y

$\Rightarrow$ (x, y) $\in \theta_a$

Therefore $\theta \subseteq \theta_a$.

On the other hand for any x $\in$ A, (a $\wedge$ x, x) = (a $\wedge$ x, 1 $\wedge$ x) $\in \theta$ **(**since (a, 1) $\in \theta$**)**

Now (x, y) $\in \theta_a \Rightarrow$ a $\wedge$ x = a $\wedge$ y

We have (a $\wedge$ x, x)$\in \theta$, (a $\wedge$ y, y) $\in \theta$ and a $\wedge$ x = a $\wedge$ y $\Rightarrow$ (x, y) $\in \theta$

Therefore $\theta_a \subseteq \theta$.

Thus θ = θ_a.

Also from (0, a)$\in \beta$ and (a, 1)$\in \theta$ we have that (0, a$^\sim$)$\in \theta$ and (a$^\sim$, 1)$\in \beta$ and by interchanging θ and β in the above argument we get that

$$\beta = \theta_{a^\sim} = \beta_a = \{(x, y) \in A \times A / \ a \vee x = a \vee y\}$$

We have already proved that a is unique with this property.

2.5. Theorem: Let A be a Pre A*-algebra with 1 induced by Boolean algebra. Then any factor congruence on A is balanced.

Proof: Let β is a factor congrucnces on Pre A*-algebra A and θ another factor congruence on A and $\theta^\sim$ a direct complement of θ.

Then by 2.4. Theorem there exist a, b$\in$A such that $\beta = \beta_a$ and $\theta = \theta_b = \beta_{b^\sim}$ and $\theta^\sim = \theta_{b^\sim}$

Now $(\beta \vee \theta) \cap (\beta \vee \theta^\sim) = (\beta_a \vee \beta_{b^\sim}) \cap (\beta_a \vee \beta_b)$

$$= \beta_{a \vee b^\sim} \cap \beta_{a \vee b}$$

$$= \beta_{(a \vee b^\sim) \wedge (a \vee b)}$$

$$= \beta_{a \vee (b^\sim \wedge b)}$$

$$= \beta_{a \vee 0} = \beta_a = \beta$$

Thus β is balanced.

2.6. Theorem: If θ is a factor congruence on A then $\theta \cap (A_a \times A_a)$ is a balanced factor congruence on A_a for each a$\in$A and there exists unique $s_a \in$ B(A_a) such that $\theta \cap (A_a \times A_a) = \theta_{s_a}$ = {(x, y) $\in A_a \times A_a$ / $s_a \wedge$ x = $s_a \wedge$ y}

Proof: It follows from 2.3, 2.4 and 2.5 theorems

Conclusion

This manuscript makes it possible to identify a factor congruence and its is a unique direct complement on a Pre A*-algebra. The notion of balanced congruence was initiated. It is detected that any factor congruence on Pre A*algebra is balanced. For any element a in a Pre A*algebra, it has been derived a typical Pre A*algebra A_a. It is observed that if θ is a congruence on a Pre A* algebra A, then there is $\theta \cap (A_a \times A_a)$ is a congruence on A_a, for each $a \in$A and if θ be a factor congruence on a Pre A*-algebra A, then $\theta \cap (A_a \times A_a)$ is a balanced factor congruence on A_a for each a$\in$A and there exists unique $s_a \in$ B(A_a) such that $\theta \cap (A_a \times A_a) = \theta_{s_a}$ = {(x, y) $\in A_a \times A_a$ /

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$s_a \wedge x = s_a \wedge y\}$

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